

Longevity Risk

Modeling, Valuation and Hedging

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Master thesis for the title of actuary

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Longevity risk

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Longevity risk : valuation

Longevity risk : hedging

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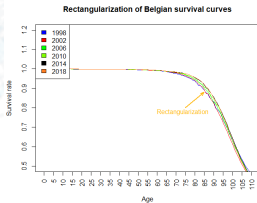
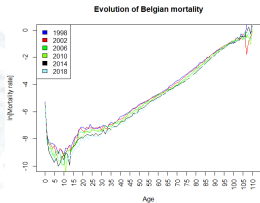
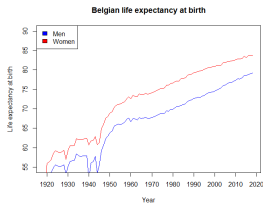
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Ultimate age and table closure
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Lee-Carter model : prospective life table
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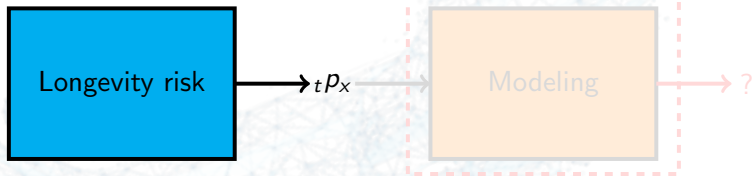


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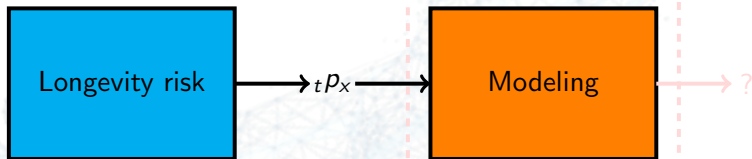
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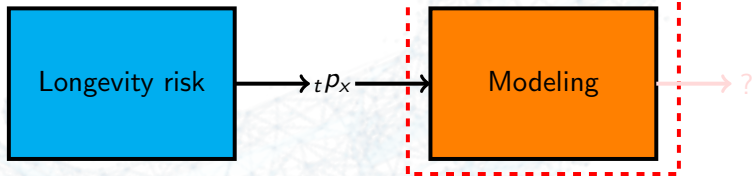


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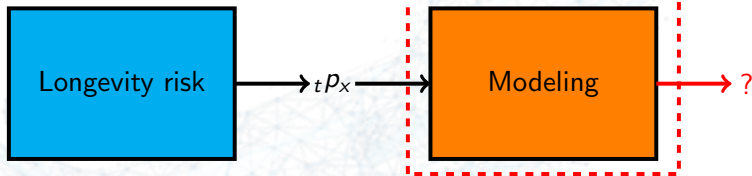
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Static life table

The goal is to compute

$${}_t p_x = \exp \left(- \int_0^t \mu_{x+s} ds \right) \quad (1)$$

Historically, first attempts were

$$\mu_x^{\text{De Moivre}} = \frac{1}{\omega - x} \quad (2)$$

$$\mu_x^{\text{Gompertz}} = \alpha c^x \quad (3)$$

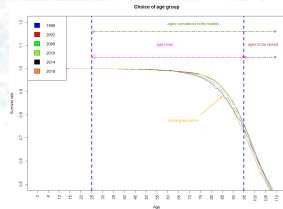
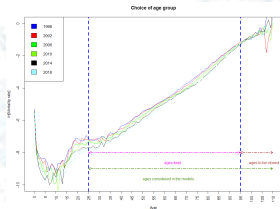
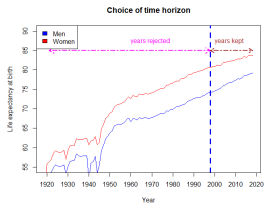
$$\mu_x^{\text{Makeham}} = A + \alpha c^x \quad (4)$$

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Choice of time horizon and age group



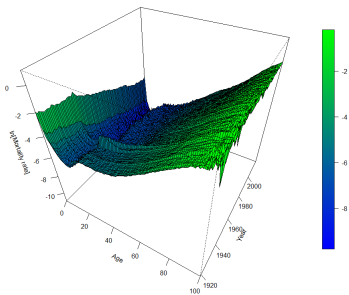
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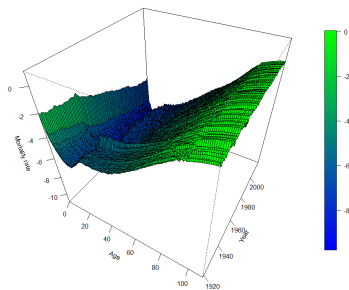
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Ultimate age and table closure

Belgian gross mortality rates per age and per year



Belgian gross mortality rates per age and per year, after table closure



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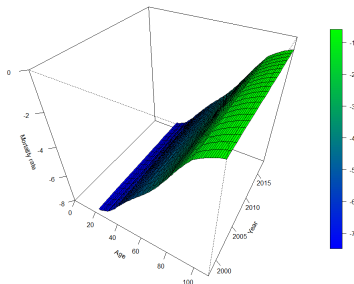
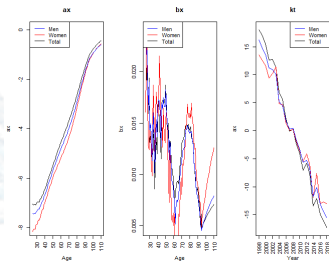
Longevity risk : modeling

Lee-Carter model : foreword

The Lee-Carter equation is given by

$$\log \mu_x(t) = \alpha_x + \beta_x \kappa(t) + \epsilon_x \quad (5)$$

Belgian gross mortality rates per age and per year, smoothed by the Lee-Carter model



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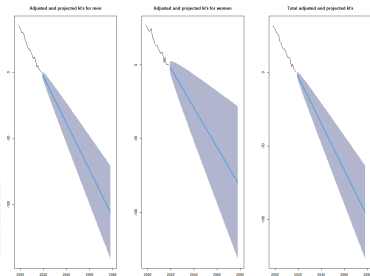
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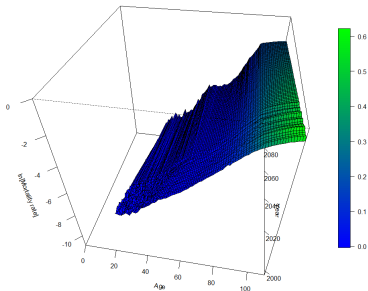
Feller model

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Lee-Carter model : prospective life table



Belgian LC prospective life table, from 1998 to 2078



Longevity risk : modeling

Feller model

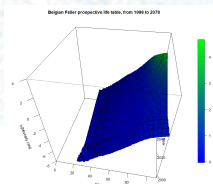
The Feller equation is given by

$$d\mu_x(t) = a\mu_x(t)dt + \sigma\sqrt{\mu_x(t)}dW(t) \quad (6)$$

MLE gives the following results for the parameters calibration :

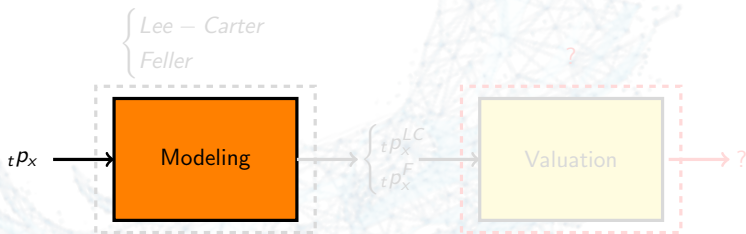
a	σ
0,03865	0,0000005144

Then, we get



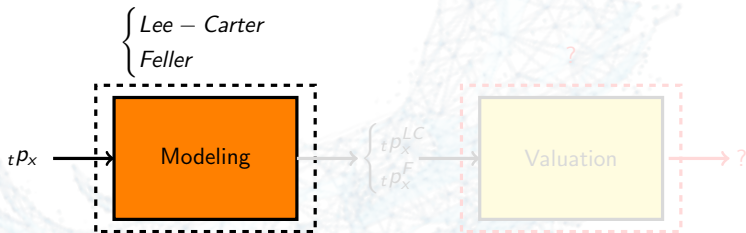
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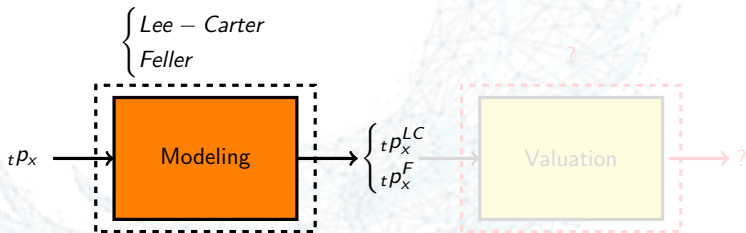
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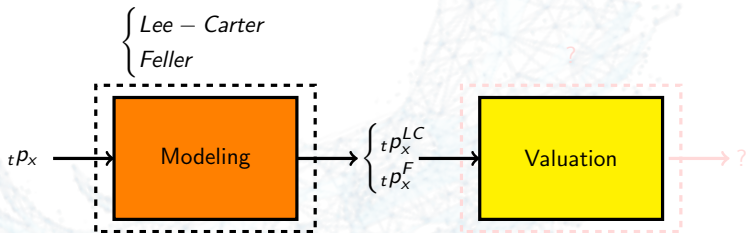
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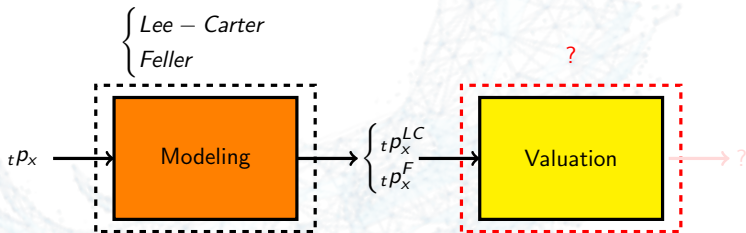
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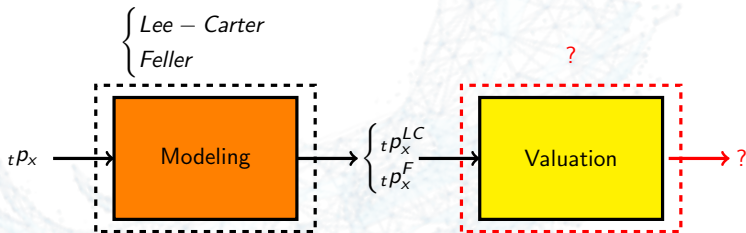
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Longevity risk : valuation

Wang transform : foreword

We apply

$${}_t p_x^* = \Phi \left(\Phi^{-1} ({}_t p_x) + \lambda \right) \quad (7)$$

with

$$\ddot{a}_x^\lambda(t) = \sum_{t \geq 1} (1+i)^{-t} \Phi \left(\Phi^{-1} ({}_t p_x) + \lambda_x(t) \right) \quad (8)$$

$$\ddot{a}_x^\lambda(t) = \sum_{k=0}^{\omega-x} (1+i)^{-k} {}_k p_x^{\text{market}} \quad (9)$$

$$p_{x+k}^{\text{market}} = p_{x+k-5}^{\text{MR}} \quad (10)$$

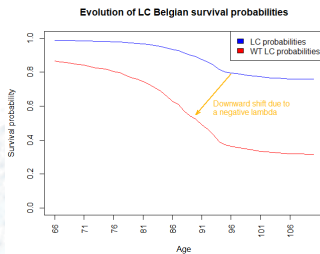
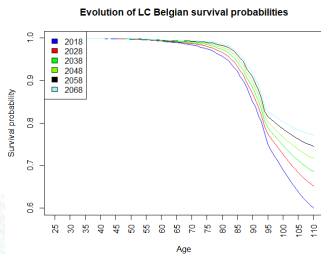
$$p_x^{\text{MR}} = s g^{c^x(c-1)} \quad (11)$$

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 Wang transform : **Lee-Carter case**
 Wang transform : Feller case

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Wang transform : Lee-Carter case



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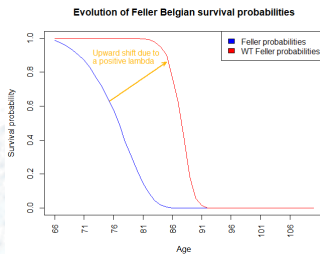
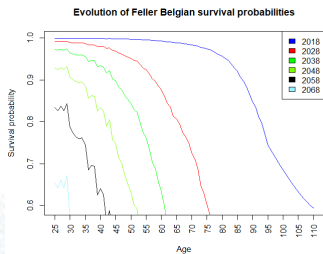
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Wang transform : Lee-Carter case

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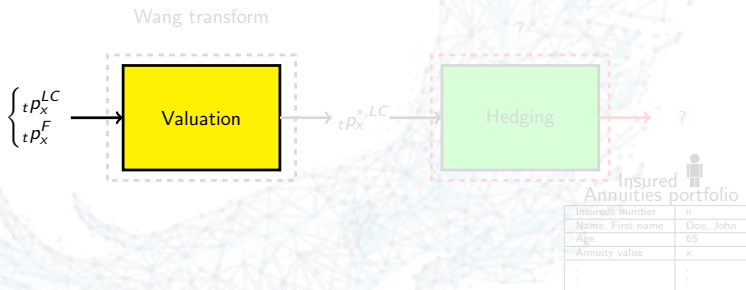
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Longevity swap and longevity bond
 Risk premium and cost of swap
 Safety margin and cost of bond

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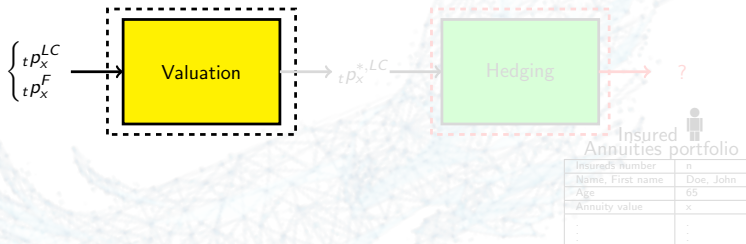
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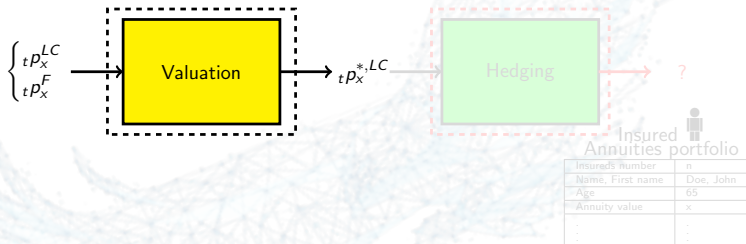
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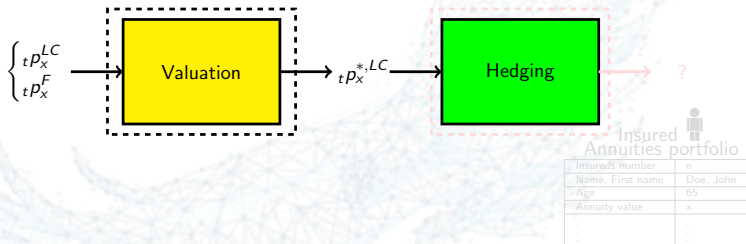
Wang transform



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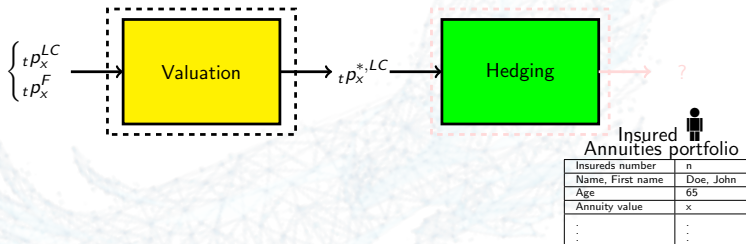
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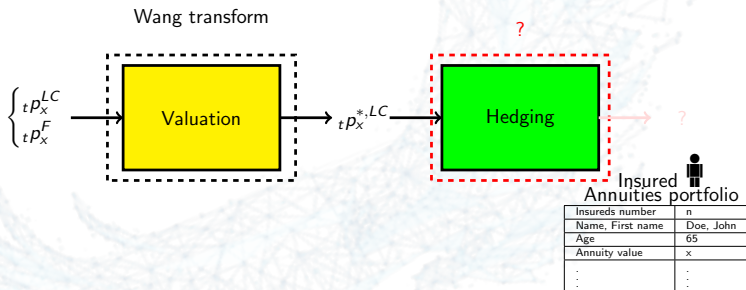
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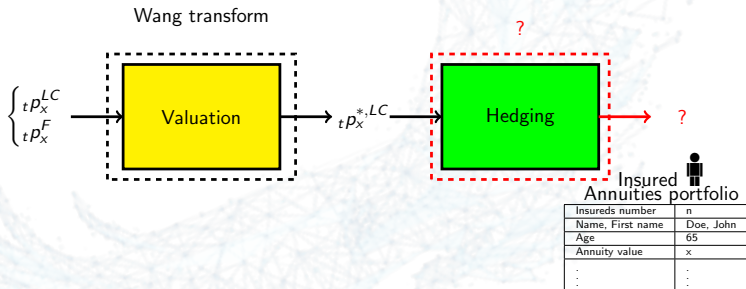
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Longevity risk : hedging

Longevity swap and longevity bond

Longevity swap

In a longevity swap transaction, periodic fixed payments are made to the swap counterparty (or (re)insurer) in exchange for periodic payments based on the difference between the actual and expected pension or annuity mortality experience.

and

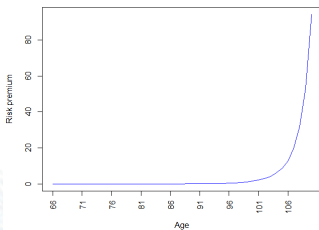
Longevity bond

The payout on longevity bonds depends on the longevity experience of a given population, so that the payment is related to the number of survivors in the population. Basically, it would pay out more as the proportion of survivors in the reference population rises.

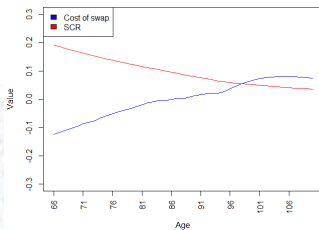
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Risk premium and cost of swap

Risk premium of the longevity swap in the LC case



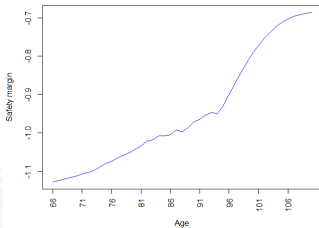
Cost of swap vs SCR in the LC case



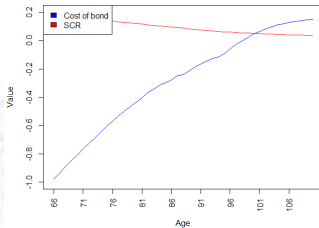
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Safety margin and cost of bond

Safety margin of the longevity bond in the LC case



Cost of bond vs SCR in the LC case



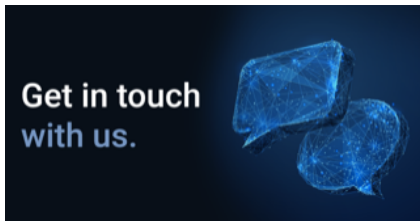
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